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International Journal of Multiphase Flow 29 (2003) 579–601

International Journal of Multiphase Flow

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Numerical simulations of random suspensions at finite Reynolds numbers ☆

E. Climent [a,*], M.R. Maxey [b]

[a] Institut de Mécanique des Fluides et des Solides de Strasbourg, 2 Rue Boussingault, Strasbourg 67000, France
[b] Center for Fluid Mechanics, Turbulence and Computation Division of Applied Mathematics, Brown University, Providence, RI 02912, USA

Received 4 April 2002; received in revised form 26 January 2003

Abstract

The sedimentation of solid spherical particles in an initially quiescent fluid is investigated through numerical simulations. Under conditions of Stokes flow, the results provide good agreement with experiments for the mean settling velocity while the velocity fluctuation levels grow with the size of the system in accordance with theoretical predictions for homogeneous suspensions. New results on finite Reynolds number suspensions are presented that illustrate the role of wake-induced interactions between particles. A significant reduction in the average settling velocity is explained by an enhancement of a wake-induced scattering process. Anisotropy in the fluid flow is evident and the evolution of velocity correlations is investigated for various particulate Reynolds numbers. Increases in Reynolds number tend to decrease the integral length scale of the fluctuating flow field and reduce significantly the Lagrangian time scale of the velocity fluctuations. Analysis of the evolution of fluctuation levels when the domain width grows indicates that inertial screening dramatically reduces the divergence.

Keywords: Sedimentation; Numerical simulation; Multi-body interactions; Wakes; Inertial screening

1. Introduction

The description of hydrodynamic interactions in dispersed two-phase flows is of great interest for the design of many industrial processes. As an example, mixing of powders in a reactive

☆ This paper was presented in the 4th International Conference on Multiphase Flow (ICMF-2001). The ICMF-2001 took place in New Orleans, USA during the week of May 27 to June 1, 2001 and was attended by 630 delegates representing 46 countries. Professor E.E. Michaelides was the chair of the conference.

* Corresponding author.

doi:10.1016/S0301-9322(03)00016-8

solvent is omnipresent in chemical engineering. Such configurations are commonly encountered too in environmental sciences: settling of micro-organisms (plankton) and migration of micron-scale particles is widely studied in oceanography, pollutant transport in underground water resource and dusty gas ejection from industrial chimneys are strictly controlled to define safety areas.

Most of the problems involved in the prediction of such complex hydrodynamic interactions arise from the extremely large scatter of length scales present in the flow. Due to an inverse cascade of energy transport, energy supplied at the smallest scales can induce large-scale motions in the fluid. Even at low volume concentration (few percent), the motion of monodisperse spherical solid particles in a suspension cannot be predicted by homogeneous theory due to non-random, multi-body interactions. From the wake-induced flow between two nearby particles to the creation of clusters of particles whose dynamics are mainly controlled by large-scale collective effects, the succession of interactions provides momentum transfer at all scales. Numerical simulations of the motion of both phases (liquid and particles) provide a valuable tool to investigate the evolution of settling characteristics.

Beyond the applications to engineering, the mean settling velocity in a suspension and the relative velocity fluctuations are of particular interest from a fundamental point of view. For low Reynolds number flows (Stokes approximation), numerous theoretical, experimental and numerical results are available (see Davis, 1996 for a detailed review). The evolution of the mean settling velocity for a monodisperse suspension is not a subject of controversy. On the other hand, Caflish and Luke (1985) predicted a divergence of the velocity fluctuations of the suspension with increasing size of the container. Most experiments do not exhibit such a divergence and fluctuation levels saturate for larger containers (see Nicolai and Guazzelli, 1995; Segrè et al., 1997). In recent investigations into the dynamics of Stokes suspensions, arguments involving non-homogeneity due to vertical concentration gradients (Mucha et al., submitted to J. Fluid Mech., 2002; Dance, 2002) or long-range screening related to sidewalls (Brenner, 1999; Ladd, 2002; Bernard-Michel et al., 2002) have been proposed. In the case of finite Reynolds numbers, most of the questions are still open. What is the impact of the loss of fore-and-aft wake symmetry on the average settling velocity and the fluctuations? The purpose of the present paper is to provide new results on random homogeneous suspensions at finite Reynolds number using two-way coupling simulations. The particles are considered non-Brownian and experience only hydrodynamic interactions when settling under gravity. We consider only periodic boxes for the simulations in order to avoid any additional factors from non-homogeneity.

The present paper is organized as follows. Particular features of the force-coupling method (FCM) used to simulate the two-phase flow dynamics are briefly outlined. Results on settling in Stokes suspensions are presented and compared to theoretical and experimental evidence in the literature. New results on finite Reynolds suspensions are then given and the associated flow structure is investigated.

2. Two-way coupling approach

Direct numerical simulation of dispersed two-phase flows is clearly difficult with present computing resources, even with the notable successes of Hu (1996) and Johnson and Tezduyar

(1996). A major difficulty lies in imposing the no-slip condition on the freely moving particles. Such simulations generally require the use of a time-dependent mesh that evolves, following individual particles. Different approaches have been taken by Glowinski et al. (1999), Patankar et al. (2000) and by Esmaeeli and Tryggvason (1998), where particles surfaces are tracked on a fixed mesh. All these simulations are limited so far to relatively low numbers of particles due to the need to resolve the flow at the particle surface, and so restricting the range of applications. Indeed the length scales involved in dispersed two-phase flows are spread between the smallest scales in the wake of each particle to the largest related to the geometry of the vessel. In a Stokes flow, the velocity field induced by the motion of a single sphere can be represented by a multipole expansion. When two or more spheres are settling, the hydrodynamic interactions may be split into a far and a near-field flow. Brady and Bossis (1988) took advantage of this length scale discrimination to develop the method of Stokesian dynamics. The far-field hydrodynamic interactions are represented by the first few multipoles and the corresponding induced flow while lubrication forces are included in a pair-wise fashion to represent the near-field effects. Improvements have been made recently by Sierou and Brady (2001) but limitations remain as to the number of particles that can reasonably be simulated and the method is restricted to Stokes flow.

Thus, approximate models are needed for the effects of the motion of finite size particles on the fluid flow. At first order, the particles induce velocity perturbations that may be represented by momentum source terms added to the Navier–Stokes equations. When the assumption that the particles are smaller than all relevant length scales of the flow is valid, each particle can be modeled by a point-force. This procedure has been successfully used to study turbulence modulation induced by collective effects of the dispersed phase in gas–solid flows (Elghobashi and Truesdell, 1993). But this kind of model is not appropriate to investigate hydrodynamic interactions between particles encountered in sedimenting suspensions. The basic idea of a point-force model is that the velocity perturbation induced by an individual particle is negligible with regard to the collectively induced modifications. This type of model is unable to account for the direct hydrodynamic interactions between individual particles.

In the FCM introduced by Maxey et al. (1997), the momentum source is no longer a Dirac function but is spread on the numerical mesh by using a finite-sized envelope with a spherical Gaussian distribution. The entire domain is filled by an incompressible Newtonian fluid, see (1) and (2), including the volume occupied by the particles. In Eq. (2), $\mathbf{f}(\mathbf{x},t)$ represents the momentum distribution related to N particles centered at $\mathbf{Y}_i(t)$. Fourier-spectral methods on a fully periodic three-dimensional domain permit the numerical solution of Eqs. (1) and (2) on a uniform grid.

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}(\mathbf{x},t) \tag{2}$$

$$\mathbf{f}(\mathbf{x},t) = \sum_{i=1}^{N} \mathbf{F}\Delta(\mathbf{x} - \mathbf{Y}_i(t)) \tag{3}$$

$\mathbf{F}\Delta(\mathbf{x})$ is a spherically symmetric force distribution function (4) and the width of the Gaussian envelope is related to the particle radius a.

$$\Delta(\mathbf{x}) = (2\pi\sigma^2)^{-3/2}\, e^{(-r^2/2\sigma^2)} \quad \text{with } r = |\mathbf{x}| \tag{4}$$

The velocity of the ith particle is determined by the local spatial average of the fluid velocity field as

$$\mathbf{V}_i(t) = \int \mathbf{u}(\mathbf{x}, t)\Delta(\mathbf{x} - \mathbf{Y}_i(t))\, d^3x \tag{5}$$

In contrast to the point-force models, the particle motion is based directly on the local velocity of the total flow field without the need for a separate model for Lagrangian tracking of the particles. Using the FCM for an isolated particle, Maxey and Patel (2001) demonstrated analytically that the settling velocity obtained by the filtered velocity (5) matches the Stokes velocity when the width of the Gaussian envelope is specified as $\sigma = a/\pi^{1/2}$. The present model has been validated for fixed cubic arrays and fixed random arrays of particles in Stokes flow by Maxey and Patel (2001). Obviously, such results are only approximations of the true Stokes flows since no-slip conditions on the particle surface are not explicitly imposed, but at distances of $a/2$ or greater from the particle surface the flow is accurately represented. The settling velocity of a pair of particles, aligned vertically or horizontally, is fairly well predicted by the present model when the distance between the particles is greater than half a radius.

The ratio $\sigma = a/\pi^{1/2}$ is also acceptable for finite Reynolds numbers with sufficiently good accuracy (Dent, 1999; Lomholt et al., 2002; Liu et al., 2002). Using the finite envelope force-coupling model to compare vorticity contours in the wake of a single particle, we find satisfactory accuracy with Reynolds numbers less than 10. Improved precision can be achieved with a higher-order multipole expansion of the coupling term (see Lomholt and Maxey, 2003). Inclusion of a force dipole in the model increases accuracy in the representation of particle–particle or particle–wall hydrodynamic interaction when the separation is less than one radius. The force dipole (6) is added to the force monopole in (3)

$$\mathbf{F}_{\text{dipole}} = \mathbf{G}\frac{\partial}{\partial \mathbf{x}}\Delta'(\mathbf{x}) \tag{6}$$

and the length scale of the finite-sized envelope $\Delta'(\mathbf{x})$, similar to (4), is set with a different width $\sigma' = a/(36\pi)^{1/6}$.

G is a second order tensor; the anti-symmetric part is related to the torque on the flow and the symmetric part (stresslet) arises if a local strain-rate is experienced by the particle. An iterative scheme is used to enforce a zero strain-rate for the averaged fluid flow located inside the spherical region of the fluid occupied by the particle. For finite Reynolds number situations, tests on fixed array of particles have successfully demonstrated that the force-coupling model is able to reproduce wake effects (Dent, 1999; Lomholt et al., 2002). Moreover, including force dipole terms Lomholt (2000) obtained the expected dynamics for particle motion close to a wall. For the simple case of two freely evolving particles, hydrodynamic interactions modeled by the FCM induce the well-known "drafting, kissing and tumbling" scenario (Lomholt, 2000; Lomholt et al., 2002). In the context of finite Reynolds flows, Liu et al. (2002) has predicted accurately both the drag and the lift forces experienced by a small particle in a Poiseuille flow using the FCM.

In (3), **F** denotes the force transmitted to the fluid by the ith particle. This comprises of the external body force due to gravity, less any buoyancy force from the fluid, and the inertia of the particle. As the fluid fills the entire domain (including the particles), the simulated particles have the same inertia as the carrying fluid. In the absence of any forces due to gravity, a particle with the same density as the surrounding fluid will not transmit any force to the fluid. The presence of the particle is then instead represented by the higher order dipole terms, as described by Lomholt and Maxey (2003). The force balance for a particle of mass m_p is

$$(m_p - m_f)\frac{d\mathbf{V}}{dt} + \mathbf{F} = (m_p - m_f)\mathbf{g} \tag{7}$$

where m_f is the mass of the volume of fluid occupied by the particle and $\mathbf{g}$ is the acceleration due to gravity. The first term in (7) represents the excess inertia of the particle relative to that of the displaced fluid. In general $\mathbf{F}$ balances the drag, added-mass, lift and history forces generated by the motion of the particle through the fluid. In a sedimenting suspension at low to moderate Reynolds numbers the particle accelerations are not large, and generally much smaller than the acceleration due to gravity. Therefore as a first approximation we may neglect the particle acceleration term in (7), $\mathbf{F}$ is then kept constant equal to the net force of gravity acting on the particle. The contribution of the excess of particle inertia will only be a small perturbation to the motion. We did monitor the particle accelerations throughout the computations and verified the approximation in all the presented results.

The particle Reynolds number Re is defined in terms of the particle diameter and the particle velocity. In any experiment the net force due to gravity, and hence $\mathbf{F}$, is known in advance while the particle velocity must be determined. The force $\mathbf{F}$ may be characterized by a non-dimensional, force Reynolds number, $Re_F = \rho F/\mu^2$. Using a standard drag law, we have a relation between Re_F and the standard Reynolds number Re for a single isolated particle settling under gravity. For example in a Stokes flow, we find $Re_F = 3\pi Re$. In the results presented, the nominal reference value of Re is the value for an isolated particle settling with the given value of Re_F, which is fixed for a set of experiments. In all the simulations, the particle radius is kept constant to enforce the same resolution of the fluid flow whatever Re number ($0 < Re < 10$). A particle diameter is roughly equal to between five and six grid points. The terminal settling velocity for a single particle is obtained from the numerical model and is used as the reference velocity scale for that Re_F. The velocity has nearly the same as the value predicted by the standard drag law. A small discrepancy still exists due to the periodic boundary conditions used in the simulations, the settling velocity of a single particle will be denoted V_0.

As the particle force Reynolds number or the particle concentration increases, the relative level of the particle accelerations increase as compared to $\mathbf{g}$. The average particle concentration c is defined in terms of the domain size L and particle radius a as

$$c = N\frac{4\pi}{3}\left(\frac{a}{L}\right)^3$$

If the density of a particle is twice that of the fluid, so that $(m_p - m_f)$ is equal to m_f, the typical observed ratio of dV/dt to $\mathbf{g}$ is about 4% when $Re = 10$ and $c = 12\%$. This represents the largest value of the ratio in the present simulations. For $Re = 1$, the ratio is less than 1% at the same concentration. For a given value of Re_F the precise value of the ratio is proportional to $(m_p - m_f)/m_f$ and the relative level of the particle acceleration is reduced if the particle density is

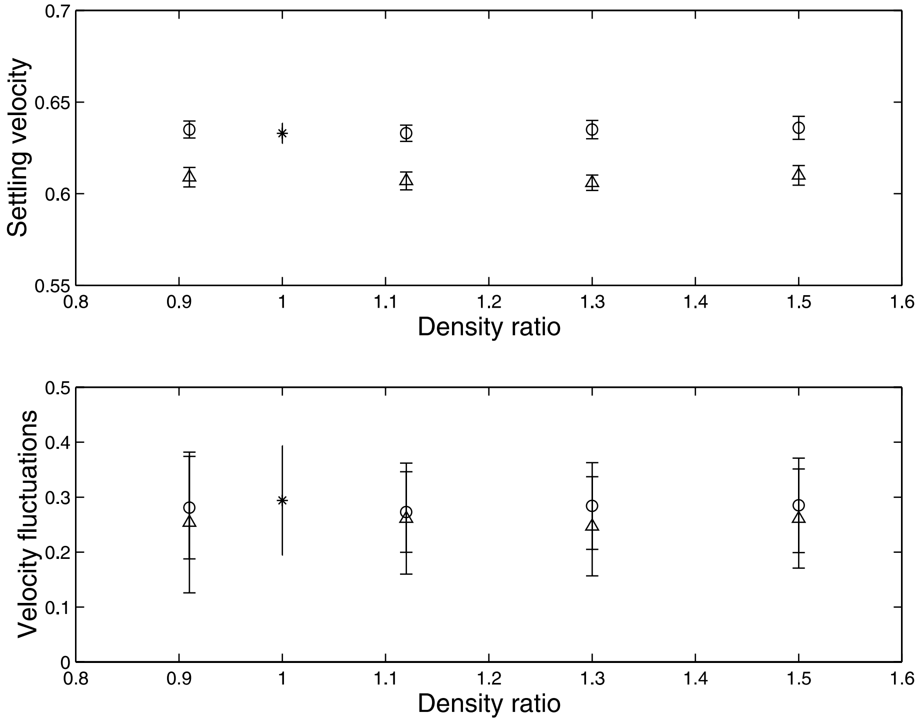

Fig. 1. Tests of dipole and inertia effects. Settling velocity and rms fluctuations are scaled by V_0. Density ratio is m_p/m_f. Error bars are related to standard deviation of the mean due to temporal evolution. ($*$) Reference case with force monopole only and without inertia; (○) force monopole only and inertia term; (△) force monopole, dipole and inertia term.

closer to that of the fluid. Thus, when the Reynolds number becomes finite, inertia is included in the model with the restriction that fluid and particles have nearly the same density.

Detailed tests of the effect of inertia and dipole terms have been done in order to validate these assumptions. A case of finite Reynolds suspension has been selected to check the evolution of macroscopic quantities (mean velocity and fluctuations) with the density ratio m_p/m_f and the order of the multipole expansion. All the results are collected on Fig. 1. The density ratio has been varied from 0.9 to 1.5, which is an upper limit for the explicit numerical scheme. Averages have been computed over more than 300 non-dimensional time units based on the velocity of a single particle and its radius. The simulations with inertia only do not show any significant changes either in the mean settling velocity or in the fluctuations. The dipole term induces a systematic reduction of averaged quantities but the maximum deviations are still small (4% for the settling velocity and about 10% on fluctuations for the worst case). This systematic response of the flow does not mean that the internal structure of the flows is significantly changed. We inspected the spatial velocity correlation functions in both simulations (monopole only or monopole/dipole). Only slight modifications have been observed indicating that the behavior of the suspension is similar in the two sets of results. Therefore, in the present configuration of free settling suspensions explicit particle inertia and dipole effects are only a small correction to the results obtained with the force monopole. Neglecting these two terms in the present paper provides an important reduction of the computing time while capturing still the macroscopic effects of fluid inertia at finite Reynolds number.

Time evolution is performed by a combination of second order Adams–Bashforth and Crank–Nicolson schemes for the motion of the fluid and the particles. The length of the cubic domain L is kept constant, equal to 2π. Various volume concentrations of the sedimenting suspension correspond to different number of particles in the domain. Potential force barriers, similar to Glowinski et al. (1999), are used to prevent unphysical overlaps of particles, as a model of contact and lubrication forces. In Stokes flow, actual contact between particles is very limited as lubrication forces rapidly slow the approach of particles. A repulsive force $\mathbf{F}_b$ is added to the force-coupling term (3) when the distance between particle centers r is less than a prescribed cut-off separation R_{ref} which is typically equal to $2.2a$. Contact duration is generally short and most of the dynamics is controlled by the force monopole itself. The force barrier is

$$\mathbf{F}_b = -\frac{F_{ref}}{2a}\left[\frac{R_{ref}^2 - r^2}{R_{ref}^2 - 4a^2}\right]^2 \mathbf{x} \tag{8}$$

where $\mathbf{x}$ denotes the separation vector between two particles. The force barrier is added in a pairwise treatment of the collision, which is consistent with Newton's third law (each particle experiences an equal and opposite force). We have performed detailed tests of different collision barriers (varying F_{ref} and R_{ref}) to demonstrate that the finite Reynolds suspension is insensitive to the microscopic description of the collisions (Dance et al., submitted to Phys. Fluids, 2003). This rough treatment of the near-field hydrodynamics of particles near contact will restrict the study to dilute suspensions (concentration lower than 15%) that is consistent with the use of the force monopole only.

3. Stokes flow assumption

Most of the studies on sedimenting solid particles have been performed for low Reynolds number flows. This particular situation corresponds to a physical configuration of small particles (10–100 μm) settling in liquids (water, glycerin or silicon oils). The density of a particle should be close to the fluid density to be consistent with the neglect of the excess inertia of a particle. Experimentally, particles could be polypropylene (density 0.91) or polyamide (density 1.12). As an example, a single particle of polyamide (radius 50 μm) settling in water corresponds to $Re = 0.05$. Larger particles could be used in more viscous fluids. Obviously, Brownian diffusion is negligible and settling characteristics are related to multi-body hydrodynamic interactions between all the particles. Inertia of fluid and particulate phases are commonly both neglected. Theoretical predictions (Batchelor, 1972) of average settling velocity provide valuable results restricted to very low concentration of uniformly distributed particles. In spite of many efforts, no evident answer has been given to the surprising theoretical divergence of the velocity fluctuations (Caflish and Luke, 1985). In this theory, a uniformly distributed suspension is exposed to fluctuations levels directly proportional to container dimensions. Although numerical simulations based on multipole expansion of Stokes flow dynamics (Ladd, 1993) or lattice-Boltzmann approach (Ladd, 1996,

1997) exhibit similar behavior, most experiments (Nicolai et al., 1995; Nicolai and Guazzelli, 1995; Segrè et al., 1997; Ham and Homsy, 1998) demonstrate no such trend. A critical issue may be the assumption that particles remain uniformly distributed. Therefore, recent theoretical approaches have proposed more appropriate alternatives such as screening mechanisms due to side-walls (Brenner, 1999) or stably stratified suspensions (Luke, 2000), as possible sources of inhomogeneity. Numerical simulations with a lattice-Boltzmann method (Ladd, 2002) indicate that vertical walls modify the fluctuations level but do not suppress the linear increase with the box size, whereas the presence of upper and lower walls leads to saturation. Presence of a vertical concentration gradient has been observed both experimentally (Tee et al., 2002) and numerically (Mucha et al., submitted to J. Fluid Mech., 2002; Dance, 2002) and is likely to be a major factor in the saturation of the velocity fluctuations. Transient evolution of this stable concentration would control the fluctuations level indicating that steady batch sedimentation is never achieved.

As low Reynolds suspensions have been extensively studied, it is an appropriate test case for the FCM. Dynamic simulations of Stokes suspensions have been performed on four mesh grids 32^3, 64^3, 128^3 and 256^3 for different particle concentrations. Initial seeding of the particles is random and ensemble averages are performed at distinct time steps, beginning from different initial conditions. After a short transient evolution (skipped in the statistics), the macroscopic quantities reach equilibrium. Averages are first made, at any instant, over all particles present in the suspension giving a temporal evolution of the mean quantities. Then a long term, time-average is formed over independent times step on a total interval of more than 1000 Stokes times t_s where $t_s = a/V_s$ and V_s is the Stokes settling velocity of a single sphere of radius a. To achieve accuracy of the temporal discretisation of the particle motion, we kept a constant time step equal to $t_s/8$ in all the simulations. Error bars added to the mean values are related to the standard deviation of the mean and indicate the level of fluctuations of mean quantities in the time series.

Analysis of the velocity fluctuations of the particles, shown in Fig. 2, indicates that this quantity increases with the relative domain size L/a, for a fixed average concentration c. This trend agrees with the theoretical prediction of Caflish and Luke (1985). Indeed, with the assumption of a homogeneous random suspension, they demonstrated that vertical velocity fluctuations scaled with $V_s(cL/a)^{1/2}$ in a domain of width L. Hinch (1988) proposed a similar scaling based on the statistical fluctuations in the number of particles locally occurring in a system with a random, uniform Poisson distribution for the particles. The statistical fluctuations may lead to a local excess number of particles on the order of $n' = (Nl^3/L^3)^{1/2}$ in a cluster or blob of scale l. A balance of viscous drag forces and excess weight or buoyancy, related to the particle number fluctuation n' in the blob, leads to a velocity fluctuation of the order of $V_S(cl/a)^{1/2}$. When the fluctuating cluster or blob size l is comparable to the size of the domain L this estimate matches that of Caflish and Luke (1985). The results in Fig. 2, where the average volume fraction c is fixed at 6%, match the scaling of the velocity fluctuations as $(L/a)^{1/2}$ in a periodic domain as L/a ranges from 12 to 96.

Segrè et al. (1997) determined experimentally characteristic length scales for the particle clusters in very large systems. If the box size is smaller than these physical length scales of the largest clusters (connected to particle radius and suspension concentration) then relative velocity fluctuations will increase with larger domain size. Comparison of the ratio between the size of the numerical domain and the particle diameter shows that our simulations are still below the characteristic length scales observed experimentally. Therefore, only the results on the 128^3 do-

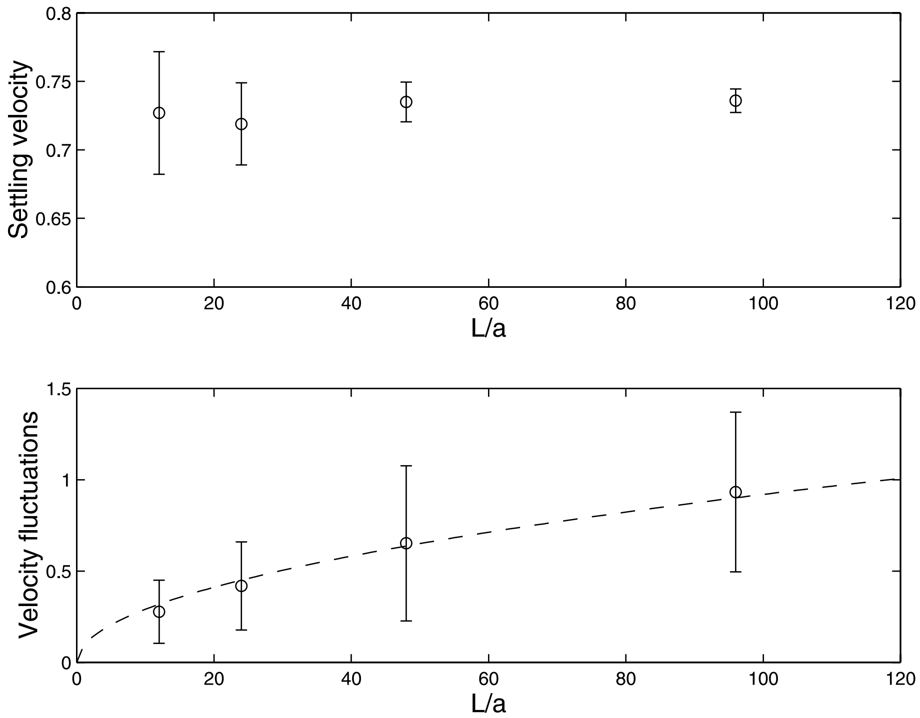

Fig. 2. Effect of the domain width on settling velocity and rms fluctuations level ($c = 6\%$, Stokes flow). Settling velocity and fluctuations are scaled by V_0. Error bars are related to standard deviation of the mean due to temporal evolution. (- - -) Scaling proposed by Caflish and Luke (1985) and Hinch (1988) $\sim [L/a]^{1/2}$.

main are presented in order to compare with the experimental data of the literature. Testing other arguments such as vertical concentration gradients and sidewalls effects are outside of the scope of the present paper. We have to note that the mean settling velocity is nearly independent of the box size (Fig. 2). A significant reduction of the error bars points out that with a larger domain the simulating box contains more eddy structures of various length scales. So the averages performed on the particles velocity at each time step are more representative of the overall evolution of the mean quantity.

Evolution of the average settling velocity with volume concentration (Fig. 3) is in good accordance with both the experimental results of Nicolai et al. (1995) and the empirical correlations of Richardson and Zaki (1954). The rms vertical velocity fluctuations (Fig. 4) are only in qualitative agreement, as our simulation domain does not capture the largest scales involved in low Reynolds suspensions. In the dilute limit, we can verify that velocity fluctuations scale with $\sim c^{1/2}$ (Caflish and Luke, 1985; Hinch, 1988). The anisotropy coefficient, comparing vertical and horizontal velocity fluctuations, points out clearly that both fluid and particles motions are dominated by the vertical velocity fluctuations associated with the gravitational settling. Vertical fluctuations are about twice the horizontal ones. Both of them have Gaussian probability density functions, which is consistent with diffusion-like dynamics.

To evaluate hydrodynamic self-diffusivities, we computed the Lagrangian auto-correlation functions $\langle V_i(t)V_i(t+\tau)\rangle$ of the temporal evolution of the particles velocity fluctuations (Fig. 5).

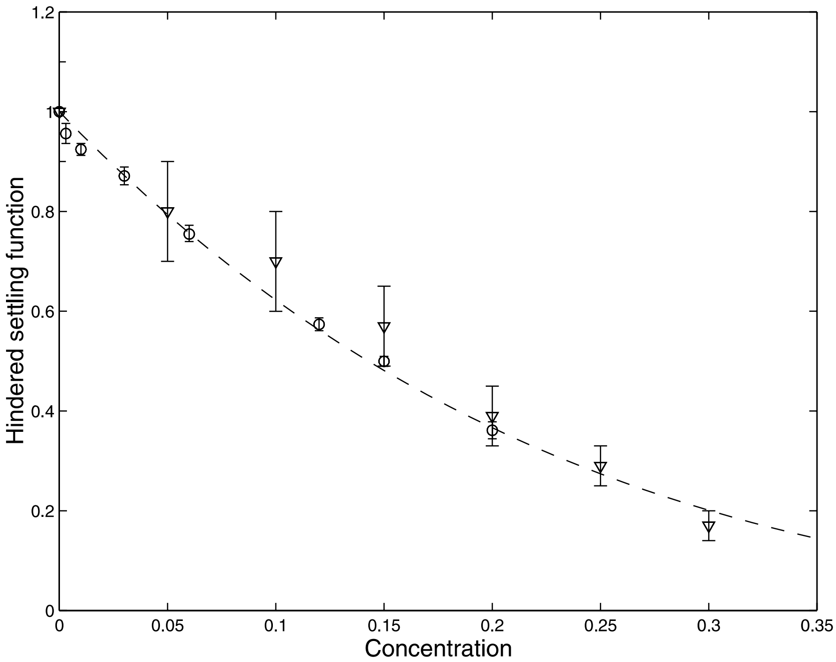

Fig. 3. Average settling velocity scaled by V_0. (▽) Experiments of Nicolai et al. (1995); (○) Stokes flow simulation on 128^3 grid; (- - -) empirical correlation of Richardson and Zaki (1954) with $n = 4.5$.

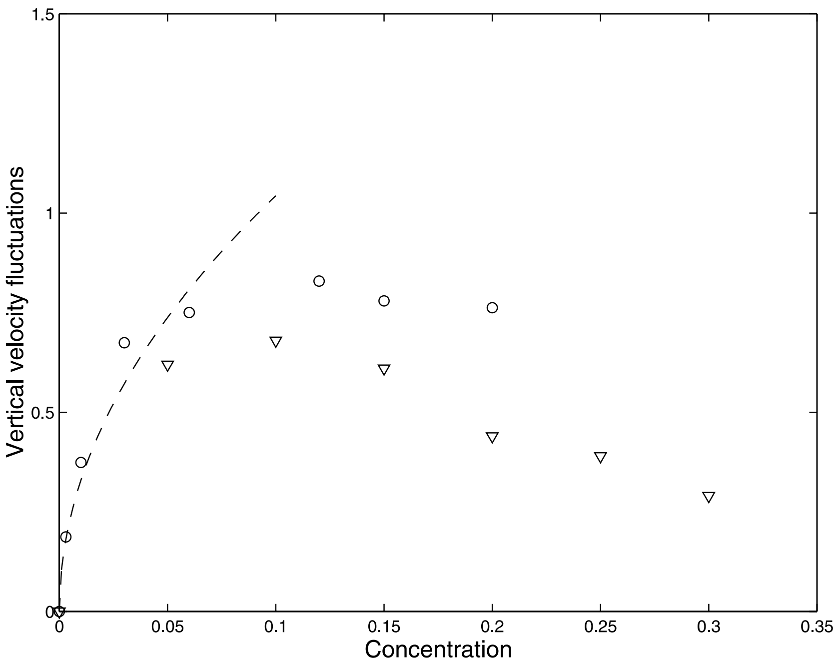

Fig. 4. Vertical velocity fluctuations scaled by V_0. (▽) Experiments of Nicolai et al. (1995); (○) Stokes flow simulation on 128^3 grid; (- - -) scaling proposed by Caflish and Luke (1985) and Hinch (1988) $\sim c^{1/2}$ for dilute suspension.

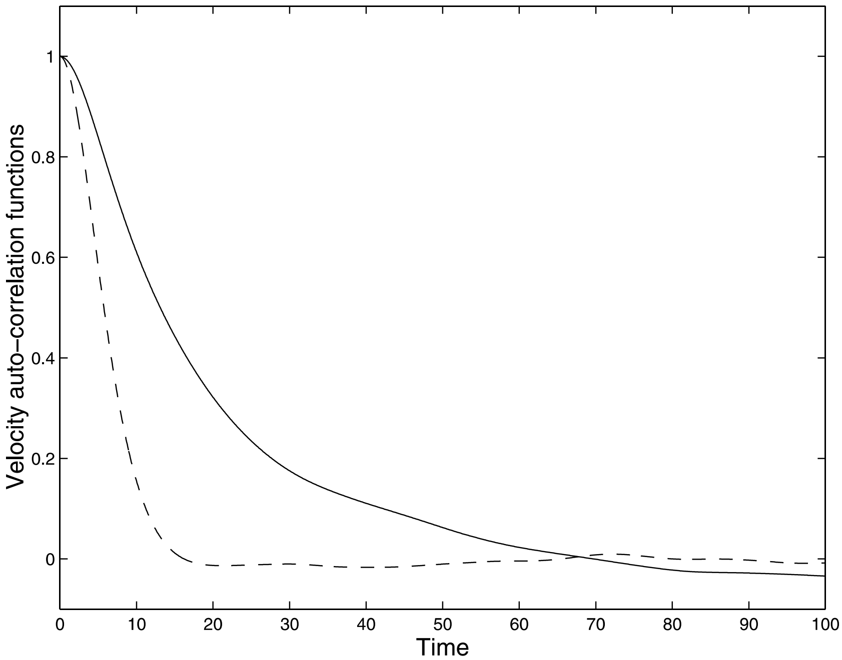

Fig. 5. Lagrangian velocity auto-correlation functions. Stokes flow and $c = 6\%$. Time is scaled by a/V_0. (—) Vertical velocity. (- - -) Horizontal velocity.

The decrease of the horizontal velocity correlation function is slightly faster than the vertical one. Thus, the velocities are uncorrelated for a non-dimensional time, scaled by t_S, ranging from 20 to 100 for different concentrations. The relaxation response of the particles velocity obtained in our homogeneous suspension is consistent with experimental results of Nicolai et al. (1995) and agree with lattice-Boltzmann simulations of Ladd (1997). Quantitative comparison of the self-diffusion coefficients cannot be achieved because the fluctuations level depends on the periodic box size (Fig. 2). Anisotropy is over predicted as obtained by Ladd (1997) with similar boundary conditions. As an illustration of velocity fluctuations in a homogeneous suspension of the particles, we present in Figs. 6 and 7 some snapshots of the velocity fields. Fig. 6 is one of the vertical planes of a 256^3 simulation. Organized eddy-like structures of various widths are noticeable as observed by previous authors (Segrè et al., 1997; Bernard-Michel et al., 2002) in PIV measurements. In a horizontal plane (Fig. 7), eddy structures seem to be even more organized, collecting the particles in large-scale spiraling motions.

As stated before, the presented Stokes flow simulations agree with other prior results on suspensions dynamics. They demonstrate that the force-coupling model used here is able to capture the principal features of large-scale hydrodynamic interactions between the particles. Of course, the assumption of periodicity for the boundary condition restricts the comparisons as most of the recent studies point out the effects of sidewalls and vertical concentration gradient. The periodic domain though is a simple geometry for which we can compare the results for Stokes flow with those obtained at finite Reynolds numbers, without introducing other factors.

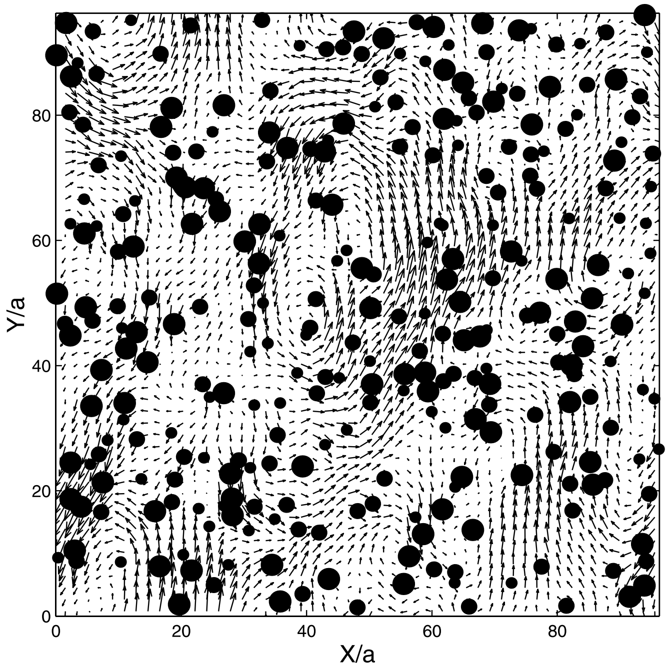

Fig. 6. Velocity field in a vertical plane for Stokes flow ($c = 6\%$ in a 256^3 simulation).

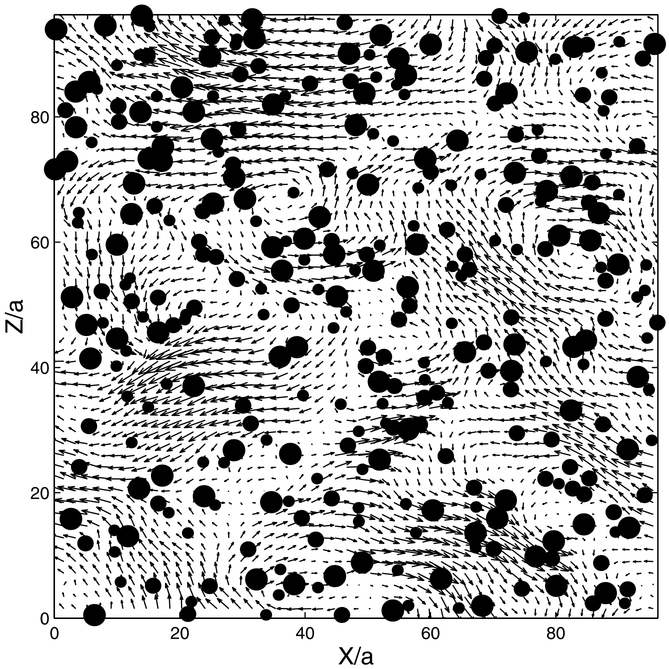

Fig. 7. Velocity field in a horizontal plane for Stokes flow ($c = 6\%$ in a 256^3 simulation).

4. Finite Reynolds number sedimentation

It is well known that the settling of two particles in Stokes flow conditions is characterized by an increase in the fall velocity while the particle separation remains unchanged. This particular feature is related to the symmetry properties of Stokes flow. Absence of inertia in the motion of the continuous phase suppresses non-linear wake interactions and gives rise to the long-range velocity perturbations that decrease slowly, inversely proportional to the separation distance between the particles. In this section we examine the impact of wakes and asymmetry in non-zero Reynolds number sedimentation. Hydrodynamic interactions take place over a shorter length scale, as fore-and-aft wake asymmetry is crucial. For example, the simple case of two particles interacting leads to the "drafting, kissing and tumbling" scenario (Fortes et al., 1987). One can expect that more complex interactions exist in random suspensions where more than two particles are involved. Relevant questions deal with the evolution of macroscopic statistical quantities with particle Reynolds number. To our knowledge, the velocity fluctuations in a homogeneous suspension have not been extensively studied in the literature theoretically, experimentally or numerically. Some interesting aspects of fluctuations level can be found in the theoretical predictions of Koch (1993) or the experimental data analysis by Cartellier and Rivière (2001).

Results on average characteristics of the suspension are presented for particulate Reynolds numbers of 0.1, 1, 5 and 10. A mean concentration of 12% involves the simultaneous simulation of $N = 3200$ particles trajectories in a periodic domain of size $L = 2\pi$ and a numerical resolution of 128 points in each direction. We remark that inertia is now present in the flow and we assume that particles have nearly the same inertia as the displaced fluid, as noted in Section 2. In these flows generated by sedimenting particles, at low to moderate Reynolds numbers, the instantaneous particle accelerations remain very small compared to the acceleration due to gravity and the excess inertia of the particles is always negligible compared to the buoyancy force. This has been monitored and verified for all the simulation results given here.

For each configuration, corresponding to fixed values of the concentration and the particulate Reynolds number, particles are initially seeded at random positions without overlap. An initial flow field is set by an Oseen approximate solution and the FCM is then used to determine the evolution of the suspension. Settling velocity and relative fluctuations are obtained from averages over multiple time steps after the transient evolution has passed and a statistical equilibrium is reached. Averages are performed on more than 300 particulate time units, based on $V_0 t/a$, where V_0 is the settling velocity of a single particle of radius a. The time step is roughly equal to 1/20 of the time unit. Time to reach statistical equilibrium from the initial condition increases with both the volumetric concentration of the suspension and the particulate Reynolds number $Re = V_0 2a/\nu$. Tests with different initial random seeding have demonstrated that initial conditions have no significant influence on the evolution of mean values. For each concentration, five simulations have been performed ranging from a Stokes flow to situations where wake asymmetry is present. Fig. 8 displays clearly the trend that the average settling velocity is appreciably reduced when the characteristic Reynolds number of the particles is increased. The settling velocity is non-dimensionalized by the free-fall velocity of a single sphere in the same domain. For moderate concentrations ($5 < c < 20\%$), best fits of V/V_0 with the Richardson–Zaki law in the form $(1-c)^n$ indicate that the exponent n decreases as Re grows. Such a trend has also been mentioned by Pan et al. (2002) in their simulations of a fluidized bed.

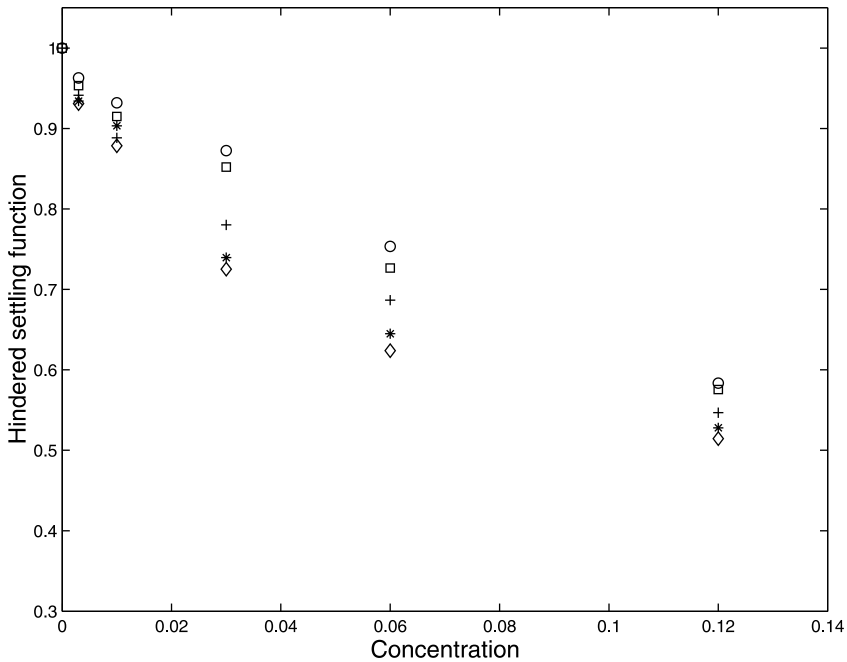

Fig. 8. Average settling velocity scaled by V_0 in 128^3 simulations: (○) Stokes flow, (□) $Re = 0.1$, (+) $Re = 1$, (∗) $Re = 5$, (◇) $Re = 10$.

The relative velocity fluctuations (Fig. 9) have the same tendency. This indicates that the variations in the vertical velocity of the particles are much less pronounced than in a Stokes flow. Comparison of vertical and horizontal velocity fluctuations indicates that anisotropy is still strong. Horizontal velocity fluctuations are nearly three times smaller than the vertical ones, indicating the influence of forcing due to gravity. As observed in Stokes flows, the probability density functions of both horizontal and vertical velocity fluctuations are strictly Gaussian. Fig. 10 shows results for the vertical velocity fluctuations.

The same anisotropy between the velocity components controls the diffusive motion of the particles whatever the particulate Reynolds number and the volumetric concentration. At low but finite Re ($Re = 0.1$ the relaxation of the temporal velocity auto-correlation functions $\langle V_i(t)V_i(t+\tau)\rangle$ shows an approximate exponential decrease (Fig. 11). The time for the horizontal velocity fluctuations to become uncorrelated is about 20 non-dimensional time units for concentrations ranging from 3% to 12%. By contrast, the auto-correlation of the vertical velocity fluctuations have a slower decrease. Therefore, the Lagrangian integral time scale is always larger for vertical fluctuations. As the vertical velocity fluctuations level is about three times the horizontal one, hydrodynamic self-diffusivities of particles are highly anisotropic. When fluid inertia is enhanced ($Re = 10$), the exponential decrease is significantly faster for both horizontal and vertical correlation functions (Fig. 12). This indicates that the diffusive motion of the particles has a much shorter time scale and the distance over which fluctuations are correlated is shorter too. This is consistent with our observation that stable statistical averages could be obtained over a shorter time interval as compared to Stokes flows. As both the velocity fluctuation levels and the

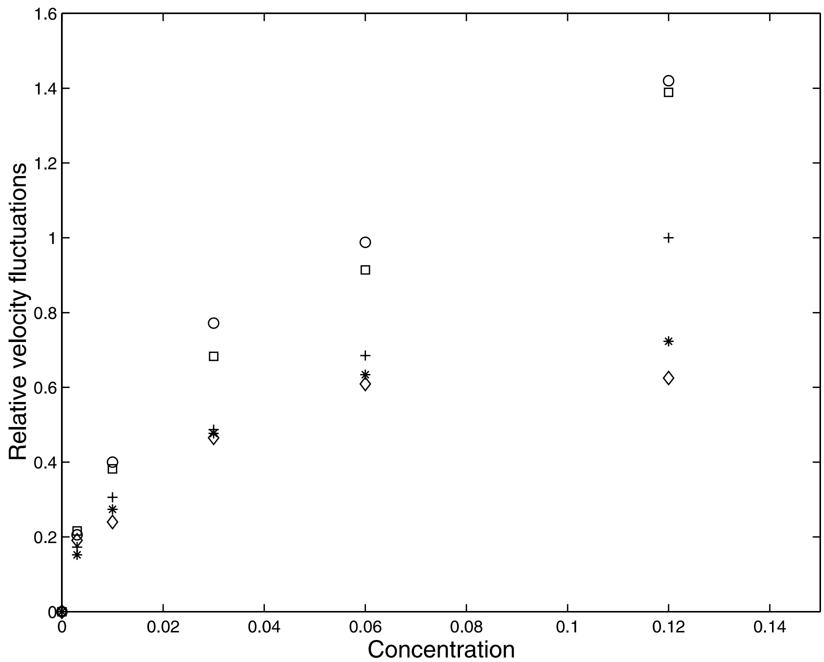

Fig. 9. Relative vertical velocity fluctuations V_{rms}/V_{mean} in 128^3 simulations: (○) Stokes flow, (□) $Re = 0.1$, (+) $Re = 1$, (∗) $Re = 5$, (◇) $Re = 10$.

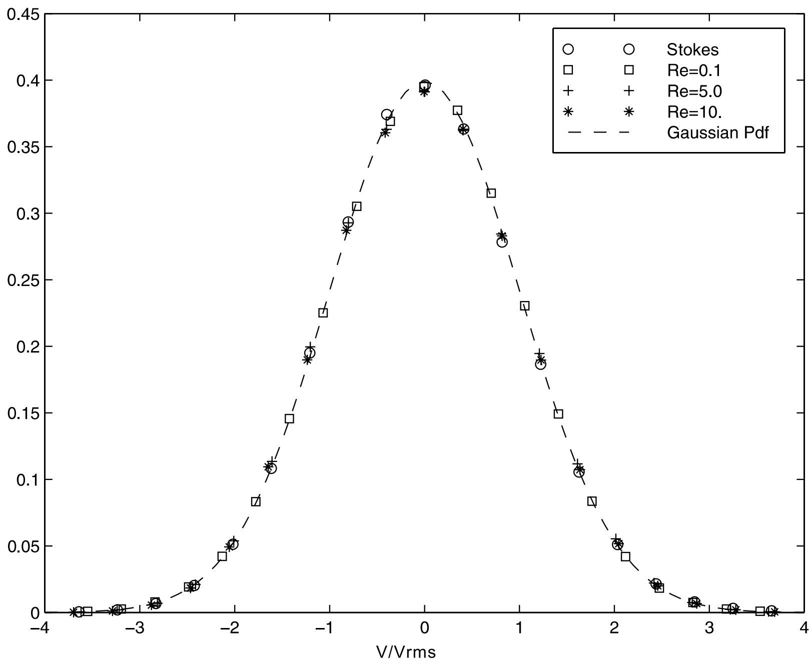

Fig. 10. Probability density function of the vertical velocity fluctuations ($c = 12\%$): (○) Stokes flow, (□) $Re = 0.1$, (+) $Re = 5$, (∗) $Re = 10$; (- - -) Gaussian distribution.

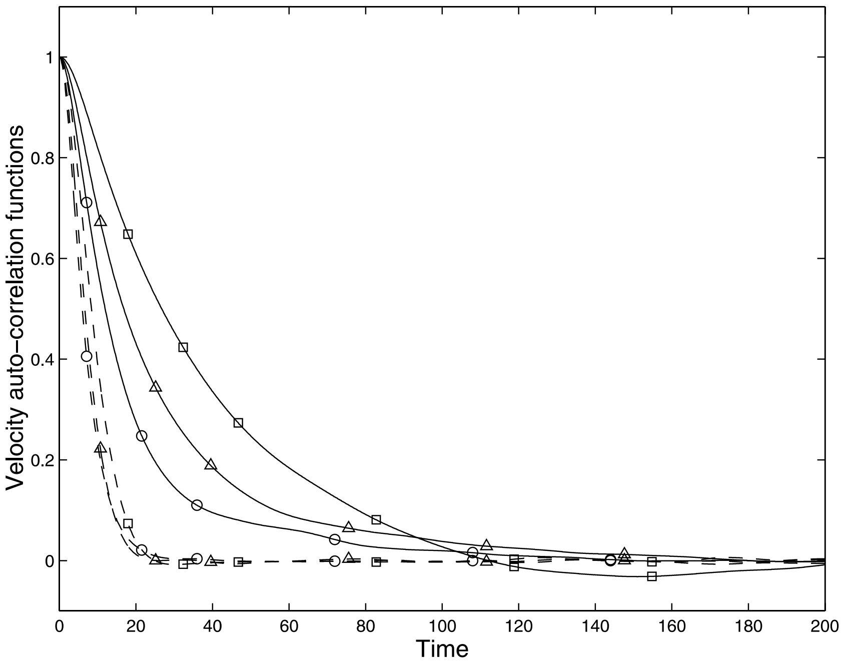

Fig. 11. Lagrangian velocity auto-correlation functions ($Re = 0.1$). Time is scaled by a/V_0: (—) vertical velocity, (- - -) horizontal velocity; (□) $c = 3\%$, (△) $c = 6\%$, (○) $c = 12\%$.

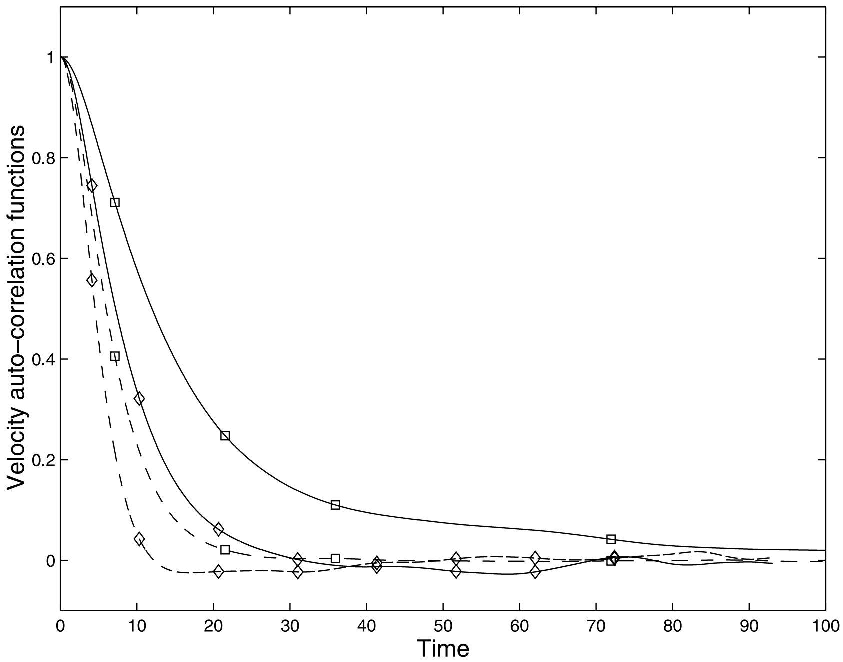

Fig. 12. Lagrangian velocity auto-correlation functions ($c = 12\%$). Time is scaled by a/V_0: (—) vertical velocity, (- - -) horizontal velocity; (□) $Re = 0.1$, (◇) $Re = 10$.

Lagrangian integral time scale decrease when Re grows, hydrodynamic self-diffusivities of the particles are drastically reduced.

Even though our quantitative results could still be size-dependent, the most critical quantity being the relative velocity fluctuations, global behavior due to Reynolds number variation should be reliable. If the propositions of Segrè et al. (1997) on the swirls and their average size are correct then direct simulations on sedimentation problems are still not possible with available computing resources due to the huge number of grid points needed for a real geometry. We checked the evolution of the settling velocity and the velocity fluctuations for a selected configuration $c = 6\%$ and $Re = 5$ (Fig. 13). We note that the mean settling velocity is nearly independent of the box size with standard deviation of the mean quantity decreasing as more samples of organized eddy-like structures are present. The most noticeable difference compared to Stokes flow suspensions is the evolution of the velocity fluctuation levels. The horizontal and vertical fluctuations were found to have the same trend. For the two largest domains $L/a = 48$ and 96, the rms value of the particles velocity fluctuation tends to saturate at a finite level. Such a behavior was suggested by Hinch (1988) and a screening mechanism induced by inertia was proposed in the theoretical analysis of Koch (1993) under the assumption of Oseen flow for the wake of the particles. Koch estimated the evolution of the velocity fluctuations in the fluid phase in terms of linearly superimposed wakes disturbances. We can guess that particles velocity should follow a similar trend. Although the evolution of fluctuation levels seems to be significantly different in the experiments reported by Cartellier and Rivière (2001), our fit of the simulation data are consistent with Koch's prediction

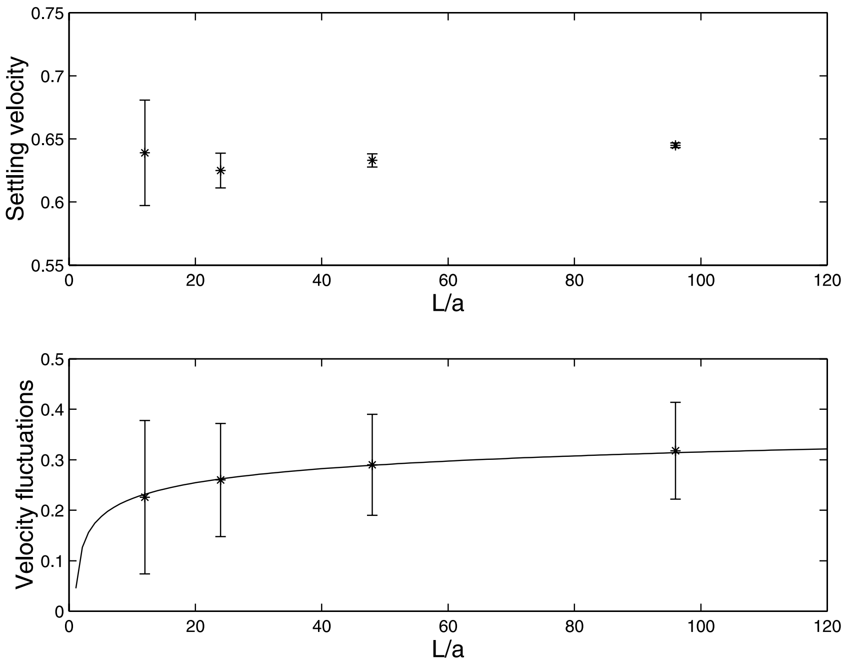

Fig. 13. Effect of the domain width on settling velocity and rms fluctuations level ($c = 6\%$, $Re = 5$). Settling velocity and fluctuations are scaled by V_0. Error bars are related to standard deviation of the mean due to temporal evolution. (—) Scaling proposed by Koch (1993).

$\sim [\ln L/a]^{1/2}$. The divergence may still exist but is considerably slower than $\sim [L/a]^{1/2}$ for Stokes flows, as discussed later.

A way to elucidate such behavior is to study statistics related to the microstructure of the suspension or relative positions of the particles. A number of statistical measures have been proposed in the literature to quantify particle dispersion in terms of pair probability density functions or repartition weighted by a second-order Legendre polynomial. Often, systematic analysis of the microstructure was performed to test theoretical arguments regarding random Poisson statistics (Lei et al., 2001) or the screening mechanisms proposed by Koch and Shaqfeh (1991). A simple relevant quantity is the average minimum distance between the particles, averaged over all the particles involved in the suspension and over several time frames. The distances are made dimensionless with regard to the typical length scale of homogeneous repartition $a(1 + c^{1/3})$. Of course, this distance decreases with increasing concentration but an interesting feature is obtained for a fixed concentration. A clear tendency of continuously increasing distance with particulate Reynolds number is observed in Fig. 14 at moderate concentration. This means that complex sequences of non-linear interactions (drafting, kissing and tumbling) enhance scattering in the relative particle positions. Therefore, the particulate phase is dispersed more evenly over the whole domain and the average settling velocity is reduced. Similar arguments can provide an explanation too for the decrease of the relative velocity fluctuations. The creation of larger clusters of particles would enhance local variations in the particles settling velocity. But as wake-induced effects tend to disperse the particles in the entire domain, the relative velocity fluctuations are reduced as Reynolds number is increased.

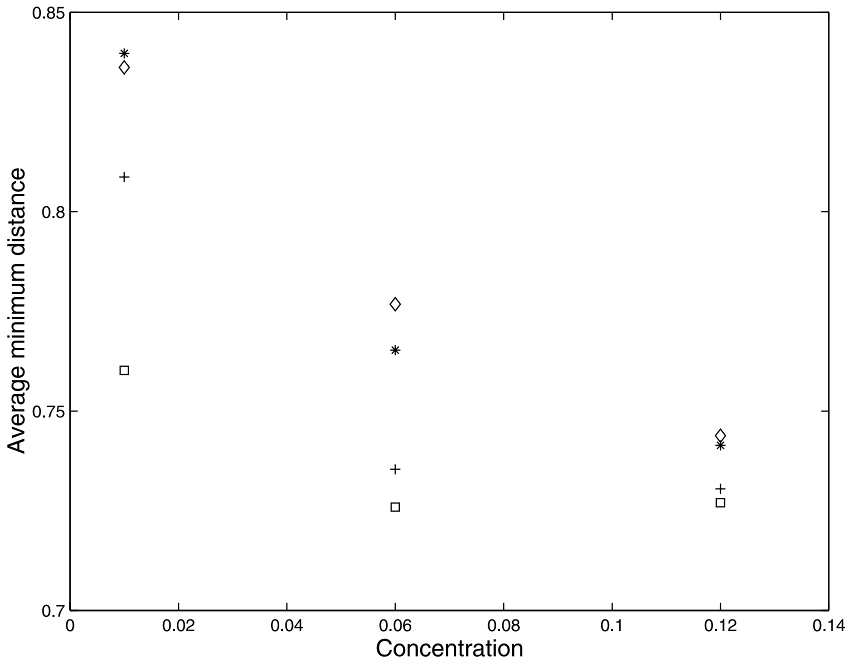

Fig. 14. Average minimum distance scaled by $a(1 + c^{1/3})$: (□) $Re = 0.1$, (+) $Re = 1$, (∗) $Re = 5$, (◇) $Re = 10$.

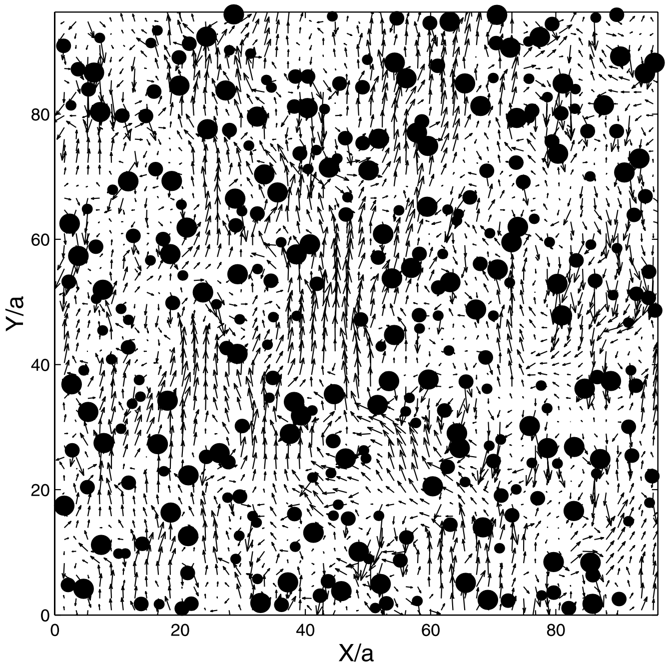

Fig. 15. Velocity field in a vertical plane for finite Reynolds flow ($c = 6\%$ and $Re = 5$ in a 256^3 simulation).

A snapshot of the velocity field and corresponding particle positions (Fig. 15) show the scattering effect of increased Reynolds number for a concentration of 6%. The selected vertical plane of the three-dimensional domain has been chosen in order to display roughly the mean number of particles. A reduction in the occurrence of clusters is related to the smaller swirls in the fluid velocity field as inertia in the continuous phase is increased, with increasing Reynolds number. The impact of fluid inertia is most important at the large scale, rather than locally, as it is too in Oseen flow theory. The particles that are not precisely centered in the selected vertical plane are shown as smaller circles based on their cross-section with the plane. Animations of the temporal evolution of the particle positions and velocity fields show clearly that collective effects are of particular importance in a sedimenting suspension.

For Stokes flow, as shown in Figs. 6 and 7, the swirls contain more particles than in finite Reynolds number flows. This is associated with a reduction of the integral Eulerian length scale when *Re* grows. In Fig. 16, the spatial velocity correlation of the vertical fluid velocity fluctuations is shown as a function of vertical separation. In each instance, the correlation decreases more rapidly as the Reynolds number increases. Typical length scales associated with swirls in the fluid velocity field are related to the spatial vertical velocity correlation. Correlations with either vertical or horizontal separations show the same trend. A first observation might be that the numerical domain is too small to avoid confinement by the periodic boundary conditions. In the best case, the domain corresponds to five integral length scales. The qualitative trends and physical explanations are undoubtedly correct even though the quantitative levels could differ from experimental data.

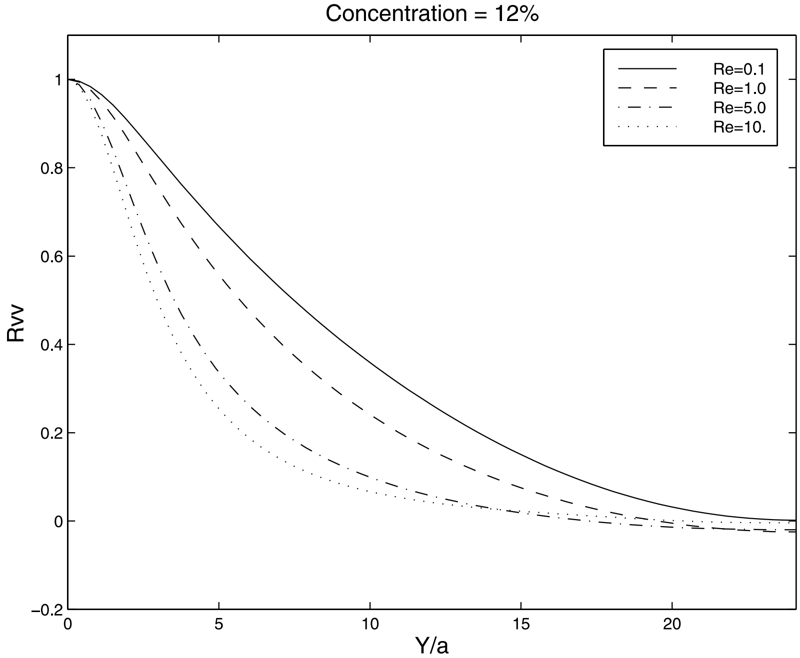

Fig. 16. Spatial correlation of the vertical velocity vs. vertical coordinate: (—) $Re = 0.1$, (- - -) $Re = 1$, (-·-·-) $Re = 5$, (· · ·) $Re = 10$.

5. Conclusion

The purpose of the present paper was to investigate the behavior of non-Brownian suspensions under low, but finite particle Reynolds number. Spherical solid particles are sedimenting in an initially quiescent fluid. Our force-coupling model of hydrodynamic interactions was first tested for Stokes flow conditions. Good agreement with the extensive experimental data and empirical correlations was obtained when both fluid and particle inertia was neglected. The level of the velocity fluctuations is found to depend on the size of the periodic domain. Scaling of these fluctuations is in accordance with theoretical predictions for homogeneous suspension in Stokes flow. Lagrangian auto-correlation functions exhibit a classical exponential relaxation and integral times scale range from 20 to 100 particulate time units, comparable to previous simulations or experiments.

New results on sedimentation are presented when fluid inertia is included in the continuous phase equations. At finite Reynolds number, non-linear interactions between two particles lead to sequences of drafting, kissing and tumbling. Due to wake asymmetry, complex interactions are encountered and our numerical simulations provide new insights on the average settling velocity and relative velocity fluctuations. A clear tendency for a reduction of both the average settling velocity and the relative fluctuations is observed when particle Reynolds number is varied from 0 to 10. A study of the correlation between these results and the internal structure of suspension is a way to elucidate 'attractive' or 'repulsive' effects of hydrodynamics interactions of the particles in random suspensions. From a comparison of the average minimum distance between the particles

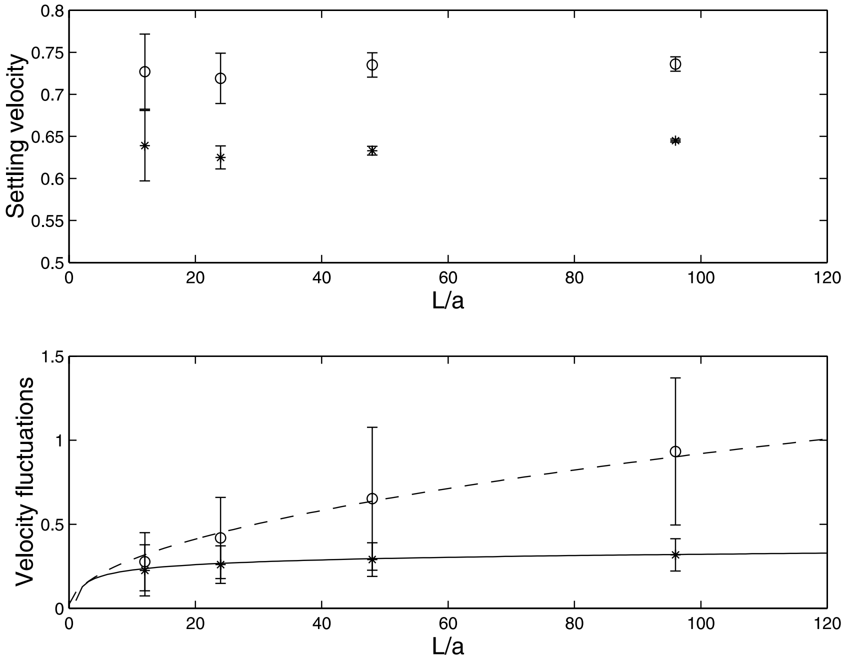

Fig. 17. Effect of the domain width on settling velocity and fluctuations level ($c = 6\%$). Settling velocity and rms fluctuations are scaled by V_0. Error bars are related to standard deviation of the mean due to temporal evolution: (◯) Stokes flow, (- - -) scaling proposed by Caflish and Luke (1985) and Hinch (1988); ($*$) $Re = 5$, (—) Scaling proposed by Koch (1993).

and the integral length scale of the velocity field, we can claim that the main effect of finite Reynolds numbers is an enhancement of particle scattering in the entire domain. Cluster formation is less pronounced when fluid inertia is included. As a result, all the particles are sedimenting with a more uniform velocity distribution and fluctuations of the settling velocity are reduced.

In Fig. 17, the domain size dependence of both Stokes and finite Reynolds suspensions are displayed together in order to highlight their very different behaviors. The mean settling velocity is nearly independent of the domain width L/a. On the other hand, vertical velocity fluctuations diverge like $\sim [L/a]^{1/2}$ for Stokes flow and tend to saturate at a finite level for $Re = 5$. The theoretical prediction (Koch, 1993) that the fluid velocity fluctuations scale as $[\ln L/a]^{1/2}$ gives a very slow variation that could easily be damped by any small experimental uncertainty in particle size or density distributions, or homogeneity of the suspension. The introduction of inertia in homogeneous settling suspension induces a screening mechanism related to the finite extent of the particles wake, the so-called inertial screening.

Acknowledgements

We are grateful to the Scientific Computing Facility at Boston University and the French Computing Centers (IDRIS–CINES) for making available computing resources. We wish to thank Dr. Sarah Dance and Dr. Suchuan Dong for valuable discussions and for their assistance with the tests of dipole terms and the parallelization of the code. Partial support by the Defense Advanced Research Projects Agency, Advanced Technology Office, under the Friction drag Program (ARPA order K042/07/39) contract number MDA972-01-C0024 is gratefully acknowledged.

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